CES-FUNCTIONS APPLICATION FOR ENSURING THE EFFICIENT SCENARIO OF INDUSTRIAL ENTERPRISE DEVELOPMENT

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Abstract. Based on the application of parameter continuation method to a forcedly homogeneous and heterogeneous CES (constant elasticity of substitution) production function in the Cobb-Douglas-Timbergen form, recommendations have been formulated regarding the general technical policy of domestic industrial enterprises at the present stage of economic development of Ukraine. Since the cost of labor resources in Ukraine is almost the lowest in the world, we can conclude that the strategic policy in the field of fixed assets of domestic enterprises claiming competitive position in the world market should be focused on replacing obsolete equipment. The application of the parameter continuation method to a forcedly homogeneous CES production function in the Cobb-Douglas-Timbergen form indicates the possibility of an extremely optimistic intensive scenario for the Ukrainian industry development, which theoretically can grow almost exceptional due to the use of skilled highly paid labor with moderate investment activity in the field of basic production tools. The use of the Cobb-Douglas-Timbergen form of inhomogeneous CES production function with the parameter continuation method made it possible to make more meaningful generalizations regarding the comparative efficiency of using the basic production factors and to give a mathematical interpretation of the outstripping growth in labor productivity law, which consists in the descending nature of the PF (production function) variation curve with respect to the elasticity coefficient of the interchangeability of labor and capital. The influence of non-stationary processes in the economy on the enterprise development scenario has been identified, due to which the "delay" effect has been revealed when a certain trend continues to occur for a certain time even though the conditions of the economic environment have been already changed. The analysis of bordering cases of the production system development based on a comparison of changes in the parameters of the production function has been carried out, a mathematical interpretation of the law of diminishing marginal productivity of the factor has been given. Recommendations on the technological policy of domestic enterprises at the present stage have been put forward.

Keywords: CES production function in the Cobb-Douglas-Timbergen form, parameter continuation method, function variation, elasticity of the labor and capital interchangeability, enterprise technical policy.

JEL: D240, C000, C010

1. Introduction

The technological policy of industrial enterprises is the basis for their competitiveness based on the fact that innovation is the most valued competitive tool in the contemporary global economics. Professionals in the field of strategic management are considering outdated equipment as one of the most threatening weaknesses of the manufacturing enterprise (Thompson, Strickland, & Gamble, 2006) and (Mital, 2013). Therefore, the approaches to the managing of the enterprise fixed assets are one of the most strategic important areas. It is clear today that the leading companies in many areas of high-tech business seek to eliminate the own production base using the third-party manufacturers which are proving much cheaper labor and industrial services, focusing purely on the innovative products development. But for domestic producers in Ukraine such an approach is not acceptable because the cost of labor in our country is already close to the global minimum, therefore these companies must look for competitive advantages in their own production base utilization. This emphasizes the issue of such a production base compliance with modern requirements. Correspondingly, it can be concluded that the strategic policy in the field of fixed assets of domestic enterprises which claim the position in the global competition should be focused on replacing obsolete equipment, which significantly reduces their competitiveness. In its turn the implementation of such a policy is complicated by the lack of financial resources. Domestic enterprises must have a solid reason behind the decision to invest in the fixed assets modernization. The use of the apparatus of production functions can provide the necessary argumentation after determining the scenario of enterprise development depending on the substitution elasticity of resources used in the production. Based on the above, the scope of the objectives of this study includes:

- the scenario identification according to which the economic system develops over a period of time;
- the impact identification of the economic non-stationary effects on the enterprise development scenario;
- analysis of theoretically possible (marginal) cases of development of the production system based on the comparison of changes in the parameters of the production function;
- development of recommendations for improving the technological policy of domestic enterprises based on the CES production functions apparatus.

2. Data and methodology

As a research methodology, the apparatus of production CES-function in the form of Cobb-Douglas-Timbergen has been used (Douglas, 1976) and (Biddle, 2020) to model the scenario of the enterprise development. The parameter continuation method was utilized to model the impact of non-stationary processes on capital adequacy and elasticity of resources substitution. The system analysis has been applied for the recommendations development concerning the technological policy of domestic enterprises.

3. Results and Discussions

3.1. Approach basics

The main approach to modeling the process was based on the production function (PF) with constant elasticity of substitution in the Cobb-Douglas-Timbergen form, which can be written as follows:

$$Y = A{\cdot}L^{\alpha}{\cdot}K^{\beta}{\cdot}e^{\omega t} \tag{1}$$

where:

Y – volume of the product

K – the amount of investment into the fixed assets used to create the volume of the product Y

A – the relationship intensity coefficient between the production factors and the volume of the product Y

L – wages that were paid during the manufacturing of the volume of the product Y

t – time

α, β – elasticity coefficients of the labor and capital interchange, which characterize the increase of output per unit of each factor increment included in the production function

ω – regression coefficient that considers the impact of the decreasing productivity law.

For completing the first task of the paper, it is necessary to compare the elasticity coefficients of labor and capital, determined by the results of the economic system over a period of time. If $\alpha > \beta$, there is an intensive scenario of economic development, if, conversely, $\alpha < \beta$ — then the economy develops according to the extensive scenario.

Data for the national industry for 1999-2019 (Statystychna Informatsiia, 2019) and the industry of the Dnipropetrovsk region for 2004-2019

(Promyslovist', 2019) have been used as an empirical basis for determining the parameters of the Cobb-Douglas-Timbergen production function.

The results of the coefficients in equation (1) calculation are given in Table 1. It is obvious that α <β for all the cases considered. Thus, for the studied periods of time the industry of Ukraine at the national level and at the level of one of the most advanced industrial regions, developed according to an extensive scenario with the small exception of the period of 2013-2017 when the wages paid in industry were significantly affected by the crisis and the significant exchange rate increase.

Table 1: *The results of the production function parameters calculation for the industry of Ukraine (2001 ÷ 2019) and the Dnipropetrovsk region (1990 ÷ 97 and 2010 ÷ 2019)*

Ukraine as a whole				Dnipropetrovskregion			
Period, years	A	β	α	Period, years	A	β	α
2004-2018	0,326	0,843	0,317	1990-1997	3,091	0,628	0,132
2005-2009	2,027	1,160	-0,335	**2010-2019**	3,422	0,438	0,348
2009-2013	2,395	0,514	0,505	2013-2019	4,750	0,893	0,232
2013-2017	4,971	0,055	0,823				

Source. Author's own contribution of the basis of statistical data.

3.2. Application of the parameter continuation method

Adopted from the calculus of variations the parameter continuation method being applied to the CES production function allows to analyze non-stationary processes preceding the moment when certain proportions of economic development, i.e., the ratio α, β and ω were established. The parameter continuation method gives a nonstationary approximation of the nonlinear functional equation and, thus, allows to identify the dynamic processes that are hidden in CES-functions in the Cobb-Douglas-Timbergen form rather conveniently. Indeed, the parameters of the PF are usually determined over a prolonged period and reflect the main trends in the labor and fixed assets utilization that have already developed over this period, but do not represent the dynamics of this scenario. Although the Cobb-Douglas-Timbergen form of the PF includes time as an independent variable, however, it just considers the influence of the diminishing productivity law, i.e., "stable and significant links between the expanded use of one of the production factors (provided all others remain unchanged) and the growth of the product" (Ershov & Sadykov, 1985). The parameter ω indicates that the economic system product grows due to qualitative changes in the technological and business processes. With negative ω according to the decreasing productivity law the scientifical and technological progress achievements implementation will lead to a decreasing return per unit of investments cost. That is, even the CES production function in the Cobb-Douglas-Timbergen form is a nonstationary with “frozen coefficients”, which corresponds to the hypothesis of the priority utilizing of one of the factors while the others remain unchanged, and thus does not consider the effects of economic development which in turn lead to the mutual influence of these factors, (Barro & Sala-I-Martin, 1992). Additionally, one of the main problems of the economy development in Ukraine is the significant lack of resources, which the pseudo-non-stationary form of production CES-function does not consider at all suffering from the absence of factors mutual influence representation.

At the same time, it is very interesting to identify non-stationary processes which affect the economic system development scenario during the period when the proportions determining that scenario had been formed. That is why we propose to utilize the parameter continuation method. It should be noted that such technique will not allow to simulate the effects associated with resources limitations but provides an opportunity to study the dynamic processes associated with changes in priorities of labor and fixed assets utilization in the production. Objectively it provides a solution to the last task of this paper, which is to form the methodological base of technological policy for the Ukrainian domestic enterprises considering the dynamic processes in the resources substitution.

To apply the method to PF in the Cobb-Douglas-Timbergen form expression (1) should be transformed to provide the strict homogeneity. This can be done by introducing the substitution of β by 1 – α. That is, expression (1) should be rewritten as:

$$Y = A{\cdot}L^{\alpha}{\cdot}K^{1-\alpha}{\cdot}e^{\omega t} \tag{2}$$

Strictly speaking, this approach brings equation (1) to the “conservative” form, i.e. artificially limits the volume of resources utilized in production and forcibly provides a situation where reducing the level of one resource utilization automatically leads to the impact increase of another. However, PF in form (2) does not consider the natural effects of limited resources that occur in the economic system — rather one can talk of the “isolated set of resources” utilization in the production process, selection of which based on a research hypothesis (Varian, 2017).

By determining the continuation of the parameter of one equation, we need to make the following transformations:

- bring expression (2) to the form $F(\alpha,t)$: $F(\alpha,t) = Y - K^{\alpha}L^{1-\alpha}e^{\omega t}$;
- identify derivatives $F'_{\alpha}(\alpha,t)$ and $F'_{t}(\alpha,t)$:

$$F'_{\alpha}(\alpha,t) = -\omega K^{\alpha}\cdot L^{1-\alpha}\cdot e^{\omega t},$$
$$F'_{t}(\alpha,t) = -e^{\omega t}\cdot K^{\alpha}\cdot L^{1-\alpha}\cdot[\ln(K) - \ln(L)];$$

- and determine the relationship $\frac{-F'_{t}(\alpha,t)}{F'_{\alpha}(\alpha,t)}$

As a result, we have:

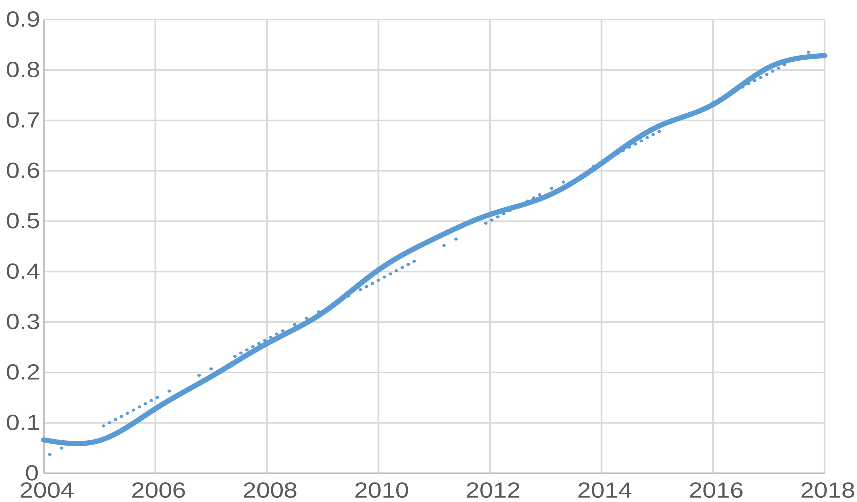
$$\frac{d\alpha}{dt} = \frac{\omega}{\ln(L) - \ln(K)} \quad (3)$$

and

$$\alpha(t) = \frac{\omega}{\ln(L) - \ln(K)}\cdot t \quad (4)$$

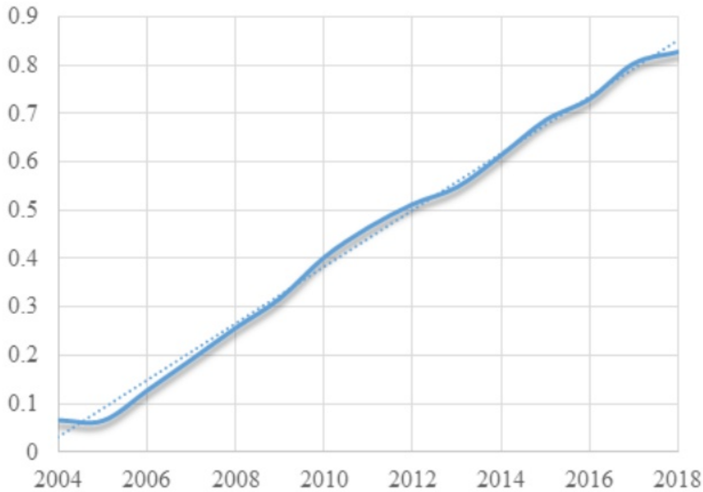
Formally, the dependence $\alpha(t)$ (4) is linear ascending (given the fact that $K>L$). But we should also consider that K and L are tabular functions of t (the characteristic form of the dependence of $\{\ln(L)-\ln(K)\}$ on t is shown in Figure 1). In fact, this is a graph of function $\frac{L}{K}(t)$ in logarithmic space. That is, the dependence $\alpha(t)$ differs from the linear one. An example of such dependence for the national industry of Ukraine for the period 2004-2018 is shown in Figure 2 compared to the linear relationship.

Figure 1: *Dependence of* $\frac{L}{K}(t)$ *for the period 2004-2018 for the industry of Ukraine in logarithmic space.*



Source: author's own contribution calculated of the basis of equation (3).

Figure 2: *Dependence of* $\alpha(t)$ *for the national industry of Ukraine for the period 2004-2018.*



Source: author's own contribution calculated of the basis of equation (4).

In addition, as it follows from equation (4) that $\alpha(t)$ can increase or decrease indefinitely over time (depending on the ratio of K and L). But we should realize that both α and ω, which are included in equation (4), in turn are functions of K and L, which determine the efficiency of each of the resource's utilization in the aggregate product. If $K>L$, then, as has been indicated already, α increases, i.e., the total product is created mainly through the efficient labor; if, on the contrary, $L>K$, then the total product is produced mainly due to extensive factors, and the dependence $\alpha(t)$ (4) is correspondingly decreasing.

In this context, it is very important to consider the order of quantities included in equation (4).

The parameter continuation method allows us to study the stability of the nonlinear equation solution (2) while postulating the hypothesis of the elasticity coefficient nonstationarity (variation) of the interchange of labor and capital α. It should be noted that in most previous works, researchers were interested in the variation of the aggregate product by resources and, accordingly, the elasticity variation (Pavlova, 2019).

In economic terms, with a strictly homogeneous production function, the curve α(t) can be identified as a neutrality trajectory, i.e., a line moving along which the smoothness of the solution surface $Y(K,L)$ is preserved. The dependence $\alpha(t)$, which we obtained as a result of the parameter continuation method utilization, is linear by shape, i.e., it indicates the absence of any singularity, bifurcation, etc. But, as already mentioned, K and L are tabular functions of t, i.e., despite the formal absence of singularities or bifurcations in equation (4), they still can occur at certain ratios of K and L.

Quite remarkable that the neutrality curve $\alpha(t)$, which was built based on the data for the Ukrainian national industry for 2004-2018, indicates an asymptotic tendency of α up to $\alpha = 0.85$, i.e. calculations by the parameter continuation method assume the possibility of a very optimistic intensive scenario for the industry development that in theory can grow almost exclusively through the utilization of the qualified high-paid labor. But this conclusion can be attributed primarily to the forced homogeneity of equation (2), which causes a deterministic mutually compensatory effect of labor on the capital utilization that may not occur in the real economy where these processes are closely interconnected but related to the dependencies being much more complicated. In addition, during the period being in our focus some contradictory processes in the field of investment were realized that have the adverse effect on the dynamics of the fixed assets value involved into the production of aggregate product — for example, the crises of 2008/9 and 2014/15 almost stopped the inflow of new investment to the fixed assets. On the contrary, rather active development of the Ukrainian economy in 2004-2007 in combination with the urgent need to modernize the national industry, contributed to the intensification of investments in fixed assets and, accordingly, to their value increase.

It should be noted that even though the fact that the PF in form (2), which was used to apply the parameter continuation method does not include the effects of mutual influence of factors, the neutrality curve built in the paper allows indirect analysis of this relationship. Indeed, expressions (3) and (4) include the difference between L and K in logarithmic space, or their ratio in metric. In turn, L and K are related to each other by the following macroeconomic laws:

- the law of outstripping growth of labor productivity and
- the law of decreasing marginal productivity of the factor.

Based on the interpretation of these laws, we can formulate different borderline cases that correspond to different configurations of the neutrality trajectory.

1. Based on the law of outstripping growth of labor productivity, we can conclude that $L \ll K$, i.e., $\alpha\ (t)$ is a strictly decreasing dependence. This, in turn, indicates that a prerequisite for the intensive scenario of economic development is the strict implementation of the law of outstripping growth of

labor productivity. This conclusion is quite trivial and can be made without rather complicated mathematical calculations and transformations. The interpretation of the law of the declining marginal productivity of the factor is of much bigger interest.

2. Due to influence of this law finally $L - K \to 0$, i.e., in the general case, the neutrality trajectory may have a singularity that is the point where bifurcation of the surface of the production function will occur. Therefore, the impact of declining marginal productivity of the factor may lead to a shift in the development scenario of the economic system. Formally speaking it is a transition from one stable regime to another.

3. If $K \ll L$, then the dependence $\alpha(t)$ is a strictly ascending dependence, i.e., in this case the degree of extensiveness increases, which usually is taken place in the Ukrainian economy throughout the whole independence period.

That is, the variation of α in t not only reflects the obvious trends in the change of α depending on K and L, but also α in turn is kind of derivative of K and L contributions to the production of the aggregate product.

In the case of inhomogeneity of the production function, i.e., in the case when $\alpha + \beta \neq 1$ in (1), to apply the parameter continuation method it is necessary to move to the system of equations by differentiating equation (1) by α, β and t and study the relationship between α and β. In this case, the system of equations for parameter continuation method will look like the following:

$$F(\alpha, \beta, t) \equiv 0; Y - K^{\alpha} L^{\beta} e^{\omega t} = 0 \tag{5}$$

$$\frac{df}{d\alpha} = -\ln(K) \cdot K^{\alpha} L^{\beta} e^{\omega t} = \Delta_2 \tag{6}$$

$$\frac{dF}{d\beta} = -\ln(L) \cdot K^{\alpha} L^{\beta} e^{\omega t} = \Delta_1 \tag{7}$$

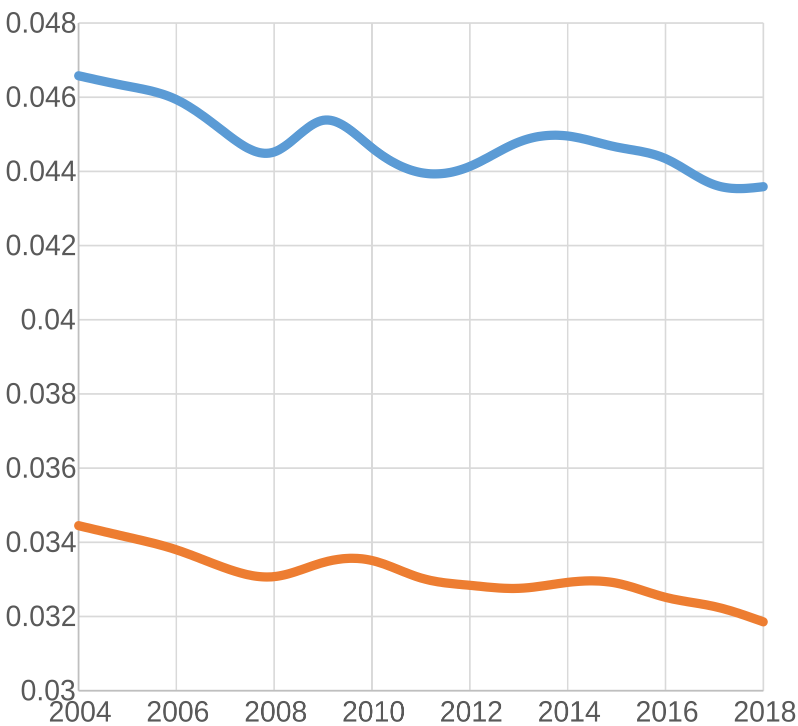
$$\frac{dF}{dt} = -\omega \cdot K^{\alpha} L^{\beta} e^{\omega t} = \Delta \tag{8}$$

$$\frac{\Delta}{\Delta_1} = \frac{\omega}{\ln(L)}; \frac{\Delta}{\Delta_2} = \frac{\omega}{\ln(K)} \tag{9}$$

That is, we obtained expressions for the variation of the production function by the elasticity coefficients of the labor and capital interchange, as well as the derivative of PF over time. To build an equations' system of parameter continuation method for inhomogeneous PF, we focus on the relations $\frac{\Delta}{\Delta_1}$ and $\frac{\Delta}{\Delta_2}$ in expression (9). Referring again to the empirical data from Table 1, we can obtain the following visualizations of the ratio $\frac{\Delta}{\Delta_1}$ and $\frac{\Delta}{\Delta_2}$ in expression (9) — see Figure 3.

It should be noted that variations of the production function on the elasticity coefficients of the labor and capital interchange are not isoquants that reflect the possibility of equivalent substitution of one factor by another. The economic meaning of variations of the production function in the Cobb-Douglas-Timbergen form is as follows — a change in the final product, which occurs due to elasticity changes of one of the factors is proportional to the regression coefficient, considering relevant factor. That is, the variations of the PF on the elasticity coefficients obtained from empirical data on the national industry of Ukraine for the period 2004-2018 indicate that the factors of production, which are interconnected by rather complex macroeconomic dependencies which are not directly reflected in the PF expression, make decreasing contribution to the total product as the intensity of their use in the production process accelerates.

Figure 3: *Graphs of functions* $\frac{\Delta}{\Delta_1}$ *(blue) and* $\frac{\Delta}{\Delta_2}$ *(orange) depending on time for the period 2004-2018 for the national industry of Ukraine*



Source: Author's own contribution calculated of the basis of equation (9).

Regarding the contribution of labor analysis, based on the results of the parameter continuation method, we can conclude the following — the results obtained quantitatively reflect the law of outstripping growth of labor productivity, the power of which is reflected by the difference values (see Figure 3, blue curve) between the end and the beginning of the study period. It should be noted that this difference cannot be interpreted as an accurate cumulative quantitative indicator of the outpacing growth of labor productivity, i.e., based on Figure 3 it cannot be said that productivity is ahead of wages by exactly 6.86%, as it follows from the graph. But the variation of the PF on the elasticity coefficient of labor exchange allows to compare trends in the manifestation of this law for different economic systems, including an individual enterprise, region, individual industry or the national economy as a whole.

After this theoretical remark it is necessary to return to the general analysis of the Figure 3. It is seen that the possible variation of PF on both parameters decreases with increasing "degree of nonstationarity" of the system, i.e., expression (8), which was obtained by differentiating over time the production function in the Cobb-Douglas-Timbergen form. The economic meaning of this observation is as follows: PF in the Cobb-Douglas-Timbergen form includes a regression coefficient ω, which considers the impact of declining productivity of alternative factors of production. The calculation of this parameter on empirical data which includes periods with significant changes in the scenarios of the economic development leads to the effect of "delay", when a certain trend continues for some time, even though conditions in the economic environment have been changed. This effect is explained by the fact that the aggregate product in the economy is usually produced due to spending incurred in previous periods. The greater the differences in development scenarios could be, the greater is this effect and, consequently, the greater is the variation, which in this case will be caused only by the most general macroeconomic dependencies, such as the above-mentioned laws of outstripping growth of labor productivity and decreasing marginal productivity of the factor.

In addition, the comparison of curves in Figure 3 indicates that the sensitivity of PF in the Cobb-Douglas-Timbergen form to changes in elasticity on labor is significantly higher than the sensitivity to changes in elasticity on capital. The economic meaning of this result, according to the author, is that during the selected period of the Ukrainian industry development only the severe crisis of 2014/15 was able to cause a formally intensive scenario of national industry development (Table 1) because with declining dynamics of the industrial production index, the practical lack of investment in fixed assets, the average wage defined in hryvnas was increased due to dramatic depreciation of national currency. Again, this is just a local effect associated with the peculiarities of the Ukrainian national industry development during the crisis. But the very approach based on comparing the sensitivity of PF to production factors, which were determined by the parameter continuation method is quite interesting, and can help clarify the subtle effects of the process of the aggregate product creation, both at the level of the production system as a whole and at the level of an individual enterprise.

Concluding about the consideration of the application of the parameter continuation method to the inhomogeneous production function in the Cobb-Douglas-Timbergen form, it should be noted again that K and L are tabular functions of time, i.e., have a strict reference to specific empirical data, and vice versa, time in (8) is considered as a continuous variable. That is, the results of the calculation of the sensitivity of PF to the elasticity coefficients, shown in Figure 3 are subjects of specific empirical data. The aim of the author was to demonstrate that the application of the parameter continuation method to inhomogeneous PF allows to analyze the manifestations of rather subtle economic patterns and evaluate their quantitative parameters related to the empirical data.

3.3. Remarks on the technological policy of domestic enterprises

Thus, the preliminary analysis based on the parameter continuation method application to the CES production function in the Cobb-Douglas-Timbergen form, allows to make the following recommendations concerning the technological policy for domestic enterprises in Ukraine at the current stage of economic development:

- investments in the modernization and renewal of fixed assets in the current period are very justified because they lead to increased competitiveness of products and at the same time do not “spoil” the development scenario of either the company or the industry as a whole;
- further increase in compensation of labor will not lead to an increase in industry intensity primarily because it was caused by inflation due to which the growth of labor productivity for a period that includes crisis years is below 10%, which is considered as the norm in developed economies;
- however, given that the sensitivity of PFs built for national industry towards the changes in labor elasticity is significantly higher than the sensitivity towards changes in capital elasticity, the urgent task for Ukrainian industrial enterprises is to introduce fairer approaches to rewarding the employees, for example, based on contribution that employees made to the production of the finished goods;

- it is clear that PF calculations are based on the assessment of value added in domestic industry without taking into account the situation on the foreign market, so companies that export their own products are in a privileged position and can implement costly projects to improve fixed assets despite the state of domestic economy;
- given the latter conclusion and the fact that reproduction in the national economy does not have an adequate source of support for depreciation, attention should be paid to the need of state policy governing the fixed assets modernization, which encourages exporting companies to invest into the equipment upgrades.

4. Conclusion

Summarizing the analysis of the CES production functions application in the development of technological policy measures of Ukrainian enterprises, the following specific conclusions can be drawn:

1) the CES production function in the Cobb-Douglas-Timbergen form provides comprehensive information on the scenario of the development of the enterprise, region, industry and even the national economy as a whole;

2) the application of the parameter continuation method to the non-stationary formulation of the CES-function in the Cobb-Douglas-Timbergen form allows to quantify the impact of such basic macroeconomic laws as laws of outstripping growth of labor productivity and decreasing marginal productivity of the factor;

3) at the same time variations of PF on the main factors of production which were received by applying the parameter continuation method do not give direct quantitative interpretation of these laws in the specific production systems just giving the chance to compare trends in the considered basic production factors utilization, and, accordingly – to predict scenarios of its development in the future;

4) the application of the parameter continuation method to the forcibly homogeneous PF in the Cobb-Douglas-Timbergen form had revealed asymptotic nature of the change in the intensity of the scenario of the economic system development;

5) by applying the parameter continuation method to the inhomogeneous PF in the Cobb-Douglas-Timbergen form the depressing effect of the “nonstationarity degree” of the system on the total possible variation of PF on both factors of production was identified;

6) based on the variational analysis of inhomogeneous PF specific recommendations concerning technological policy for the Ukrainian enterprises were formulated in the context of a current stage of national economic development;

7) as a direction of further research it is necessary to indicate the application of the developed approach to individual enterprises belonging to different fields of business and a comparative analysis of trends that can be established on the basis of comparison of PF variations on the main factors of production.

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